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Contribution from Pulsars [and Discussion]

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Contribution from pulsars†

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A detailed review of the information on the primary cosmic-ray spectrum in the energy interval 10^{12}–10^{16} eV is given. Methods based on the analysis of the secondary cosmic rays in the atmosphere are described and the results are discussed.

By using quantitative predictions based on the mechanism suggested by Gunn & Ostriker, and experimental information about pulsars, the energy spectrum of primary cosmic rays is obtained. Under the assumption that the confinement time of the cosmic rays at energies around 10^{15} eV is of the order of one million years, the energy spectrum obtained agrees with the experimental observations in the energy interval 10^{14}–10^{16} eV. Different mechanisms of accelerations are required below and above the stated energies.

1. Introduction

The cosmic-ray spectrum in the region around 10^{12} eV seems to be well described by a power law with integral spectral index about -1.75. As was pointed out by Wdowczyk & Wolfendale (1973) the continuation of the spectrum with the same index gives intensities at energies around 10^{15} eV much below those derived from extensive air shower measurements. The immediate question arises as to what extent the e.a.s. measurements can be considered as accurate, since they are based on some assumed conversion factor from size of shower to primary energy. The conversion factors at various levels of observations are dependent on assumptions about the model of high energy collisions but fortunately that dependence is very weak when we consider showers at their maximum of development and, further, it can be shown that for energy conservation reasons the factor cannot be smaller than a certain value.

2. Estimation of primary particle energy from e.a.s. measurements

As is well known, an extensive air shower is a cascade of particles in the atmosphere initiated by a primary cosmic-ray particle which in practically all cases is a nuclear active particle and most likely it is a proton. Such a particle interacting produces secondary charged pions which, together with the other produced particles and surviving nucleon, initiate a cascade of nuclear particles. Decay of the charged pions produces the muon component in e.a.s. The neutral pions decaying to γ-quanta initiate the electromagnetic component of e.a.s. When the total flux of particles is registered the electromagnetic component is the component which is usually dealt with. From that point of view the extensive air shower can be understood as the superposition of a number of pure electromagnetic cascades.

In the case of such cascades detailed calculations carried out by various workers show that at the maximum the number of electrons in the cascade is equal to the primary energy expressed in GeV, so the conversion factor is 1 GeV per particle. This value sets the absolute limit on the

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† In large part the present work is based on a paper by Karakula, Osborne & Wdowczyk 1974.

conversion factor for e.a.s., but that absolute limit is clearly lower than the more realistic limit expected for showers initiated by the interactions of nuclear active particles. In the case of such showers, first of all part of the energy is transferred to the muonic component and secondly the continuing nuclear active component makes the cascade longer and consequently reduces the size at maximum.

Typical cascade curves calculated using a standard model of high energy interactions are given in figure 1. It can be seen that in a wide energy region the conversion factor amounts to 2 GeV per particle at shower maximum. This value seems to be most reasonable and it is difficult to reduce it when proper allowance is made for various fluctuations.

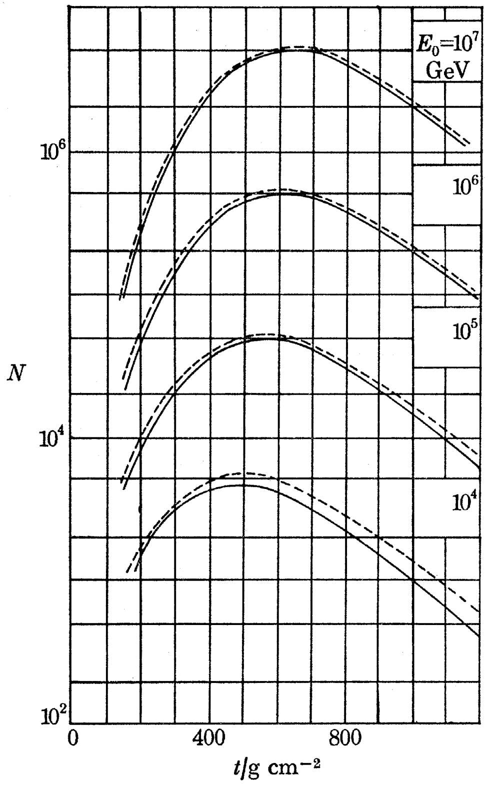

FIGURE 1. Cascade curves for extensive air showers calculated on the basis of a standard model.

3. SPECTRUM OF HIGH ENERGY COSMIC RAYS

Using the above quoted value for the conversion factor and the results of e.a.s. measurement at Mt Chacaltaya (Bradt *et al.* 1965) the lower limit for primary intensity was obtained for energies exceeding 3×10^{14} eV. The result was obtained assuming that showers reach their maximum at the Chacaltaya level for the whole considered size interval and that is the reason why the result should be taken as the lower limit. The result together with the estimation of the primary spectrum made by the Chacaltaya group and an extrapolation of the intensities around 10^{12} eV are given in figure 2. It can be clearly seen that the intensities at energies around 10^{15} eV are higher than those obtained on the basis of the extrapolation with constant slope. That fact suggests that there should exist somewhere a turn up in the spectrum. In fact the direct measurements of the total primary intensities made by Grigorov *et al.* (1971) which show good agreement with those of Ryan, Balasubrahmanyan & Ormes (1972) in the energy interval 10^{11}–2×10^{12} eV indicate some flattening of the spectrum above 3×10^{12} eV.

A more detailed analysis of the problem of primary intensities made by Kempa, Wdowczyk & Wolfendale (1974) on the basis of a variety of e.a.s. data has also confirmed the existence of the turn up in the spectrum. In that paper a conservative analysis of e.a.s. data from lower altitudes (2700–3900 m above sea level) shows the existence of an even more pronounced 'bump' on the primary spectrum. Very good internal consistency of the various experimental data is observed. Results of that analysis are shown in figure 3.

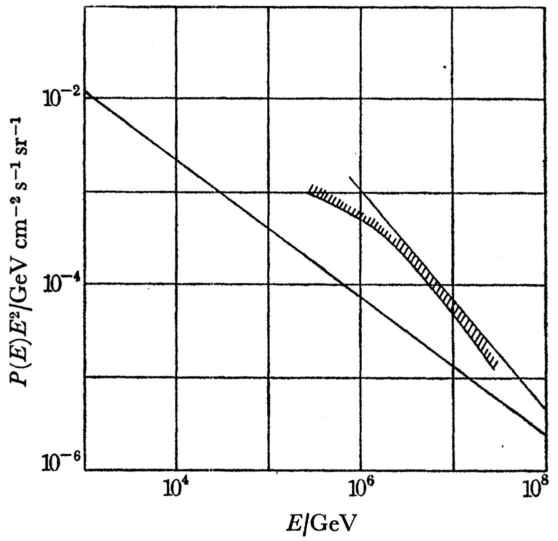

FIGURE 2. Primary energy spectrum derived from e.a.s. at maximum of their development compared with extrapolation of the directly measured spectrum around 10^{12} eV.

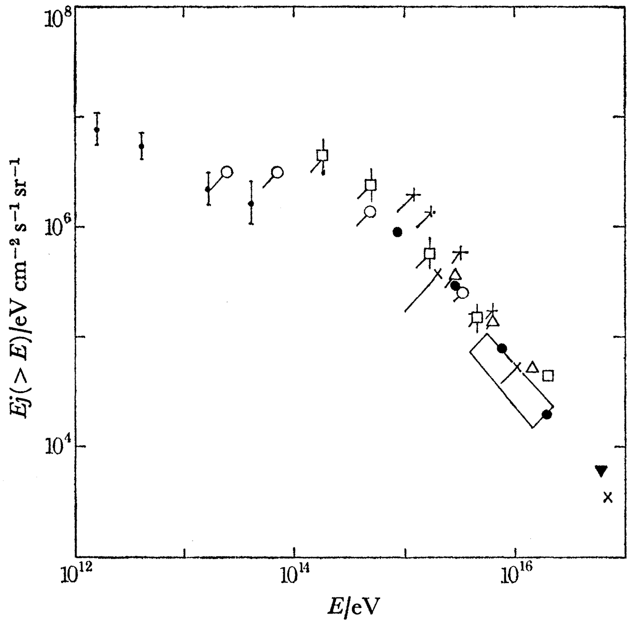

FIGURE 3. Integral energy spectrum of primary cosmic rays in the range 10^{12}–10^{17} eV. *Key*: ○, Nikolskii (1962); □, Chudakov (referred to by Nikolskii 1962); ●, Bradt *et al.* (1965); ▼, La Pointe (1968); +, Kameda *et al.* (1960); △, Miyake *et al.* (1962); ×, Khristiansen (1972 – electron size data). ◇, Lodz data (see text; upper limit $E^{\frac{1}{4}}$ multiplicity law; lower limit, $E^{\frac{1}{2}}$) (Olejniczak 1973); ⫯, Kempa (1973).

4. Possible reasons for decrease of the primary spectrum slope

In the present paper a discussion is given of possible sources of the decrease of the primary cosmic-ray slope at energies around 10^{14} eV.

One possibility would be to suppose that the cosmic rays above 10^{11} eV are produced in certain sources and that the slope fluctuates from source to source so consequently we would deal with flatter spectra at higher energies. This process could extend to a few times 10^{15} eV and then a cut off should be superimposed to obtain the observed steepening of the spectrum there. Another (although not fully different) possibility is that extra particles from sources different from those for the bulk of cosmic rays are causing the decrease of slope. Obvious candidates for the 'different type of source' are pulsars since pulsars can generate high energy particles and it is hard to expect them to accelerate low energy particles with sufficient efficiency.

Very often pulsars are associated with the possibility of accelerating protons to 10^{21} eV; this, however, requires either collapsing to much higher densities than in the case of neutron stars or particle injection with relatively high initial energies. The maximum energies of accelerated protons for more realistic parameters of pulsars are rather of the order of a few times 10^{16}–10^{17} eV. Another argument against pulsar origin of the highest energy cosmic rays is their high degree of isotropy; in the case of pulsar origin domination by our Galaxy would be very strong and a marked anisotropy would have to be expected. This arises because the contribution from pulsars from our own Galaxy should be at least two orders of magnitude higher than that from extra-galactic ones, since the frequency of pulsars should be expected to be on average proportional to the density of matter. It should be pointed out that the fluctuations in the birth of pulsars, sometimes quoted, cannot play an important role as the straggling due to deflexions in magnetic fields gives a much longer time distribution than the average time separation of the subsequent supernovae. (At $E_0 \simeq 10^{19}$ eV the straggling time should be of the order of 10^3 years as compared to 25–30 years for the separation of supernovae.)

So far there is no well elaborated theory of pulsars and in particular of the process of pulsar acceleration of particles to extremely high energies. The only theory of acceleration which can give relatively well defined quantitative predictions is that of Gunn & Ostriker (1969). That theory assumes vacuum propagation of magnetic waves and the density of the magnetosphere particles according to the suggestion of Goldreich (1969). Under those assumptions the Gunn & Ostriker theory gives the predicted spectrum as a function of parameters of pulsars which can be either measured or relatively easily estimated on the basis of the neutron star model. The theory gives the shape of the spectrum and the absolute normalization comes from the assumed density of the pulsar magnetosphere.

5. Predictions of the model

The spectrum derived on the basis of the Gunn & Ostriker model has the form:

$$\left.\begin{aligned} N(E)\,\mathrm{d}E &= 6.5\times10^{73}\,\frac{P\,\mathrm{d}P/\mathrm{d}t}{\epsilon_e^2}\,E^{-2.5}\,\mathrm{d}E, \quad \text{for} \quad E_z < E < E_M \\ &= 1.38\times10^{15}\,\frac{I^{\frac{1}{2}}}{(P\,\mathrm{d}P/\mathrm{d}t)^{\frac{1}{2}}}\,E^{-2}\,\mathrm{d}E, \quad \text{for} \quad E < E_z, \end{aligned}\right\} \tag{1}$$

where the energies are expressed in electronvolts. In the relation P is the period of the pulsar at time t, $\mathrm{d}P/\mathrm{d}t$ is its rate of change, and I and ϵ_e are the moment of inertia and elipticity. The first part of the equation corresponds to the early stages of the pulsar history when the gravitational losses dominate and the second term to the stage when magnetic losses dominate. E_z corresponds to the situation when both types of losses become equal, and is given by the expression:

$$E_z = \frac{1.31\times 10^{38}}{\epsilon_e^{\frac{4}{3}}\, I^{\frac{1}{3}}}\,(P\,\mathrm{d}P/\mathrm{d}t)\ (\mathrm{eV}). \tag{2}$$

The maximum energy E_M corresponds to the moment of the birth of the pulsar. In fact the maximum energy would not be reached in the spectrum as the particles produced at the earliest stage cannot penetrate the condensed shell of the supernova (see Barrowes 1971). The maximum energy can be expressed as:

$$E_M = 4.06\times 10^{11}\, E_z^{\frac{1}{3}}\ (\mathrm{eV}). \tag{3}$$

The spectrum of cosmic rays emitted by the pulsar can be expressed as a function of E_z in the following form:

$$\left.\begin{aligned} N(E, E_z) &= 5\times 10^{34}\left(\frac{I}{\epsilon_e}\right)^{\frac{1}{3}} \frac{1}{E_z^{\frac{1}{2}} E} \quad \text{for} \quad E < E_z \\ &= 5\times 10^{34}\left(\frac{I}{\epsilon_e}\right)^{\frac{1}{3}} \frac{E_z}{E^{2.5}} \quad \text{for} \quad E_z < E < E_M. \end{aligned}\right\} \tag{4}$$

The essential point of this picture is that at a given time the pulsar emits only particles with a given energy and the total spectrum is obtained by integrating over the whole lifetime of the pulsar.

The total energy emitted during the pulsar lifetime follows as:

$$E_{\mathrm{tot}} = \int_0^{E_M} N(E, E_z)\, E\, \mathrm{d}E = 1.5\times 10^{35}\left(\frac{I}{\epsilon_e^2}\right)^{\frac{1}{3}} E_z^{\frac{1}{2}}. \tag{5}$$

6. AVERAGE ENERGY OF COSMIC RAYS FROM PULSARS

Assuming that the elipticity and moment of inertia are the same for all pulsars (values $I = 1.4\times 10^{45}$ g cm^2 and $\epsilon_e = 2\times 10^{-4}$ which are appropriate for the Crab pulsar, were taken) the distribution of E_z for the pulsars with measurable P and $\mathrm{d}P/\mathrm{d}t$ parameters was obtained. The data from the review by Manchester & Taylor (1972) were taken. The approximation involved in using identical values of I and ϵ_e for all pulsars is not too serious in view of the lack of sensitivity of the various cosmic-ray quantities to those parameters and in any case, the expected range of values is rather small. The obtained distribution of the parameter is given in figure 4 and it can be approximated relatively well by the expression:

$$f(X) = 0.306\, X^{-0.133} \exp\,(-0.284X), \tag{6}$$

where $X = E_z/(10^{14}\,\mathrm{eV})$. The function $f(X)$ is normalized to unity $\left(\int_0^\infty f(X)\,\mathrm{d}X = 1\right)$. Assuming that the distribution obtained is representative for the population of pulsars in the Galaxy the average cosmic-ray energy spectrum normalized to one pulsar can be calculated from the convolution expression:

$$N(E) = \int_0^\infty N(E, E_z)\, f\left(\frac{E_z}{10^{14}}\right) \frac{\mathrm{d}E_z}{10^{14}}. \tag{7}$$

The energy spectrum so derived is given in figure 5. The broken curve corresponds to the spectrum obtained allowing for the absorption in the supernova shell; the absorption was estimated assuming that the mass of the shell is equal to 2.2 solar masses and that the velocity of expansion is 10^4 km/s (see Cameron 1971).

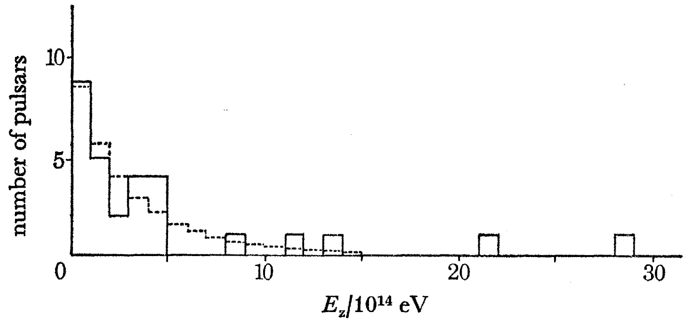

FIGURE 4. The observed distribution of pulsars as a function of breaking point energy E_z. The broken curve shows the empirical fit, $f(E_z/10^{14}\text{ eV})$, multiplied by the total number of pulsars plotted.

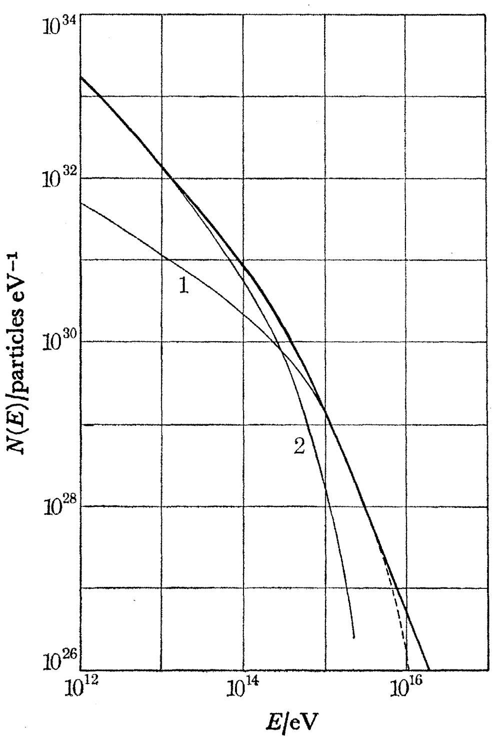

FIGURE 5. The average energy spectrum emitted per pulsar.

The total energy emitted per pulsar can be calculated from:

$$E_t = \int_0^\infty E_{\text{tot}}\,(E_z)\,f\left(\frac{E_z}{10^{14}}\right)\frac{\mathrm{d}E_z}{10^{14}} = 7.5\times10^{59}\text{ eV}, \tag{8}$$

where $E_{\text{tot}}\,(E_z)$ is given by expression 5.

From figure 5 it can be seen that the maximum energy obtained from pulsars under the mechanism discussed barely exceeds 10^{16} eV.

In order to calculate the total cosmic-ray flux it is necessary to know the birth rate of pulsars and the confinement time as well as the confinement volume. For the purpose of the present estimation it has been assumed that the frequency of supernova explosions is 1 in 26 years (after Tammann 1970) and that the cosmic rays are confined to the galactic disk (15 kpc radius and 600 pc average thickness) for a period of 10^6 years, independent of energy. This value is chosen from measurements of spallation products at much lower energies and is assumed to be valid at the energies in question here. The recent measurement on slopes of spectra of boron and nitrogen and carbon and oxygen (Balasubrahmanyan & Ormes 1973) suggests that the lifetime of cosmic rays in the Galaxy decreases when the energy increases. The decrease however cannot be very substantial since difficulties would arise in explaining the observed isotropy of cosmic rays. It seems that in order to assure the observed isotropy of cosmic rays it is necessary to assume lifetime of the galactic cosmic rays at least of the order of 10^6 years (Bell, Kota & Wolfendale 1974; Dickinson & Osborne 1974). The view adopted here is that the confinement time for the galactic cosmic rays is of order of 10^6 years and that any reduction of the lifetime in the Galaxy perhaps can be due to an admixture of extra-galactic cosmic rays. In the paper of Karakula *et al.* (1974) there is an error in that the values for the thickness of the disk and the lifetime of the cosmic rays are given as 300 pc and 2×10^6 years. The values actually used in obtaining the spectrum in that work were as given here, namely 600 pc and 10^6 years.

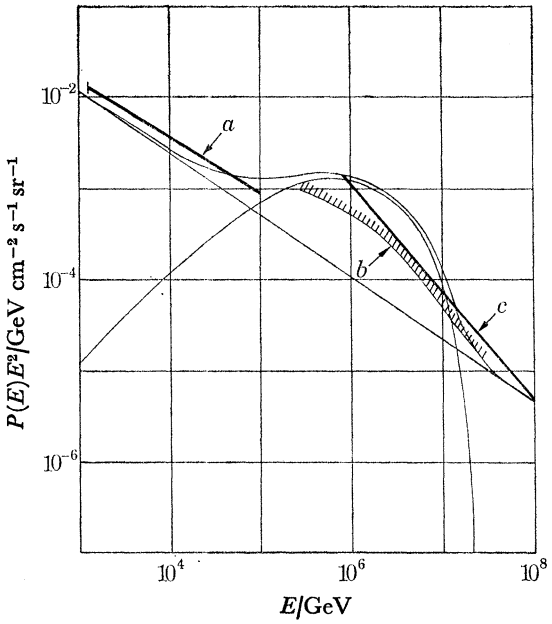

FIGURE 6. The total cosmic-ray energy spectrum obtained under the assumptions stated in the paper. The spectrum from pulsars is marked together with a spectrum with differential slope 2.7. The sum of the two is also plotted. The experimental points are taken from: *a*, indirect analysis, muon measurements; *b*, analysis of Chacaltaya e.a.s. data lower limit; *c*, e.a.s. measurements of Bradt *et al.* (1965).

The spectrum resulting from the considerations described is given in figure 6 together with a single slope spectrum obtained by extrapolation of direct measurements around 10^{12} eV (with integral slope 1.7). The composed spectrum and some experimental estimations of the primary intensities are also shown. It can be seen that the contribution from pulsars calculated on the basis of the model at energies around 10^{15} eV is the same as the intensity measured.

This fact is especially encouraging since the input information has all been taken directly from astronomical data. The composite spectrum obtained agrees very well with the experimental data collected in figures 2 and 3.

7. CONCLUSIONS

The spectrum obtained by a combination of the simple power law and the contribution from pulsars as shown in figure 6 well explains various features of the primary spectrum. In particular it explains the decrease of the slope above 10^{12} eV as suggested earlier in this paper and also in the work by Wdowczyk & Wolfendale (1973) in connexion with the interpretation of cosmic-ray data from the point of view of high energy physics. The spectrum shows also a relatively sharp kink at about 3×10^{15} eV which is such a dominant feature of the observed primary cosmic-ray spectrum. The 'kink' arises as result of rapid cut off due both to the existence of the maximum energy in acceleration by particular pulsars and to the cut off coming from the absorption in supernova shells.

The problem of the origin of the single slope partial spectrum remains outside the scope of the present considerations. If the same spectrum is contributing to the primary intensities below 10^{14} eV and above 10^{16} eV an extra-galactic origin is preferable but either possibility (galactic or extra-galactic origin) is in principle acceptable. The extra-galactic origin seems to be preferable since it is difficult to find galactic sources able to accelerate particles in such a wide energy interval and moreover it is difficult to imagine sufficiently strong confinement which would preserve the single slope. In the case of galactic origin one should expect some steepening of the spectrum above say 10^{16} eV where significant escape of the particles should take place (see Bell *et al.* 1974).

A final point which should be mentioned is the acceleration of particles heavier than protons. This is possible in the discussed mechanisms and in fact those particles can be accelerated to energies Z times higher. As a result of that fact if any significant acceleration of heavies exists they should be observed at energies exceeding 10^{16} eV, where the spectrum of pulsar accelerated protons is already rapidly falling down. In the discussed two component model the mass composition would change with energy from the observed mixed composition to dominance of protons above 10^{14} eV and then perhaps to a region of heavy mass particles at 10^{16}–10^{17} eV and again to predominantly protonic composition when the extra-galactic component takes over say above 10^{17} eV. Some experimental confirmation at least in the region around 10^{15} eV comes from measurements of fluctuations in extensive air showers. Both the fluctuations of N_μ/N_e ratio (for summary see Adcock *et al.* 1968), and the age parameter fluctuations (Catz *et al.* 1973) indicate that at 10^{15} eV protons play an important role. The possibility that above 10^{16} eV heavies are dominating cannot be ruled out at the present moment by the existing experimental data on e.a.s.

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Discussion

J. G. **Wilson** (*University of Leeds*). Shower studies at Chacaltaya, in the 10^{15}–10^{16} eV range, were outstanding pioneering work which in many respects have not been improved upon. But now, when evidence in detail in this region is becoming a critical feature, there are strong reasons for these experiments to be repeated. In the last ten years understanding of shower measurements with sampling arrays has made very notable advances, for example in the treatment of sampling and detector characteristics, core location and optimization of the parameters directly measured, and I have no doubt that more certainly based results could now be attained. These considerations probably apply also, with varying emphasis, to other measurements in this range.

A. W. **Wolfendale**. I agree completely with Professor Wilson's remark about the value of the Chacaltaya work and also about the desirability of repeating the experiments. However, it should be stressed that the spectral form referred to by Dr Wdowczyk is based not only on the Chacaltaya measurements but also those from Mt Norikura and the Pamir laboratory.